Queuing Theory For Hospital/Health Care Operations Management

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What Queuing Theory Is & Why It's Relevant To The Hospital

- **Created to mathematically analyze waiting lines; it thus facilitates an understanding of waiting**

- **Used to see effect of changing capacity; it thus facilitates capacity planning**

- **Extended to provide optimal control of waiting; it thus can be used to dynamically manage waiting**

Why Queuing Theory Is Needed

- **Consider a bank:**
  - **To which one client comes every minute on the minute to the bank every minute**
  - **In which one teller takes exactly one minute to serve each client**
- **If the first client arrives at the same time the teller arrives there will never be any delays**

Why Queuing Theory Is Needed

- Now consider a second bank:
  - To which one client comes to the bank on the average once every minute
  - In which one teller takes an average of one one minute to serve each client
- Even if the first client arrives at the same time the teller arrives there will be a queue which will slowly grow longer and longer

Why Queuing Theory Is Needed

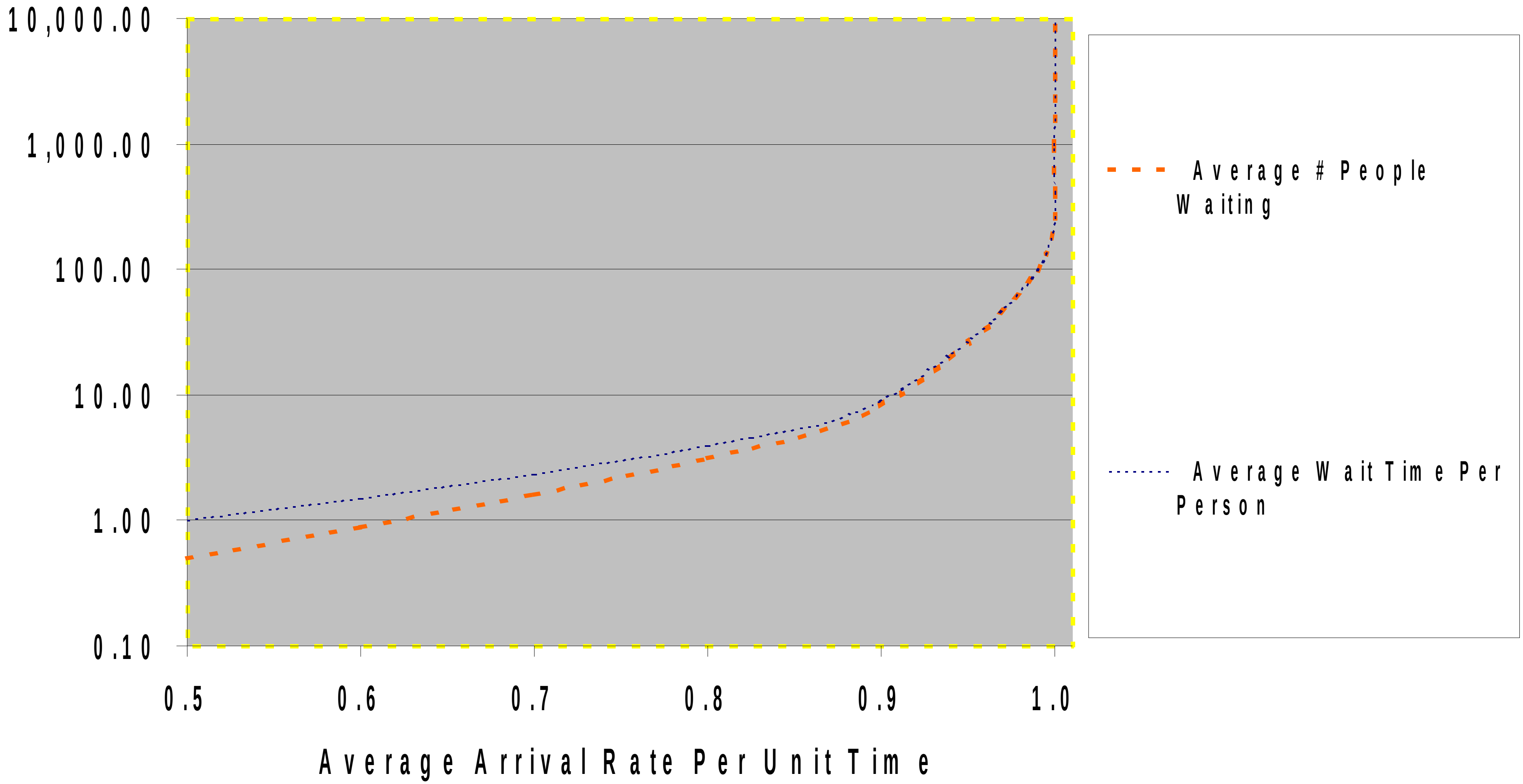
- **When there is no variability we can get 100% utilization of our tellers without any waiting at all**
- **When there is variability we need to reduce utilization and/or otherwise manage waiting**
- **With queuing theory (or simulation) we can determine the amount of waiting and the changes needed to manage the waiting**

Queuing Theory Insight – Queuing Processes Are Non-Linear

- When there is a single server (and when interarrival times and services times are both exponentially distributed), queueing statistics are very non-linear in arrival rates

Arrival Rate	Service Rate	Utilization	Average # People Waiting	Average Wait Time Per Person
0.5000	1.00	0.5000	0.5	1.0
0.6000	1.00	0.6000	0.9	1.5
0.7000	1.00	0.7000	1.6	2.3
0.8000	1.00	0.8000	3.2	4.0
0.9000	1.00	0.9000	8.1	9.0
0.9900	1.00	0.9900	98.0	99.0
0.9990	1.00	0.9990	998.0	999.0
0.9999	1.00	0.9999	9998.0	9999.0

Queuing Theory Insight – Queuing Processes Are Non-Linear

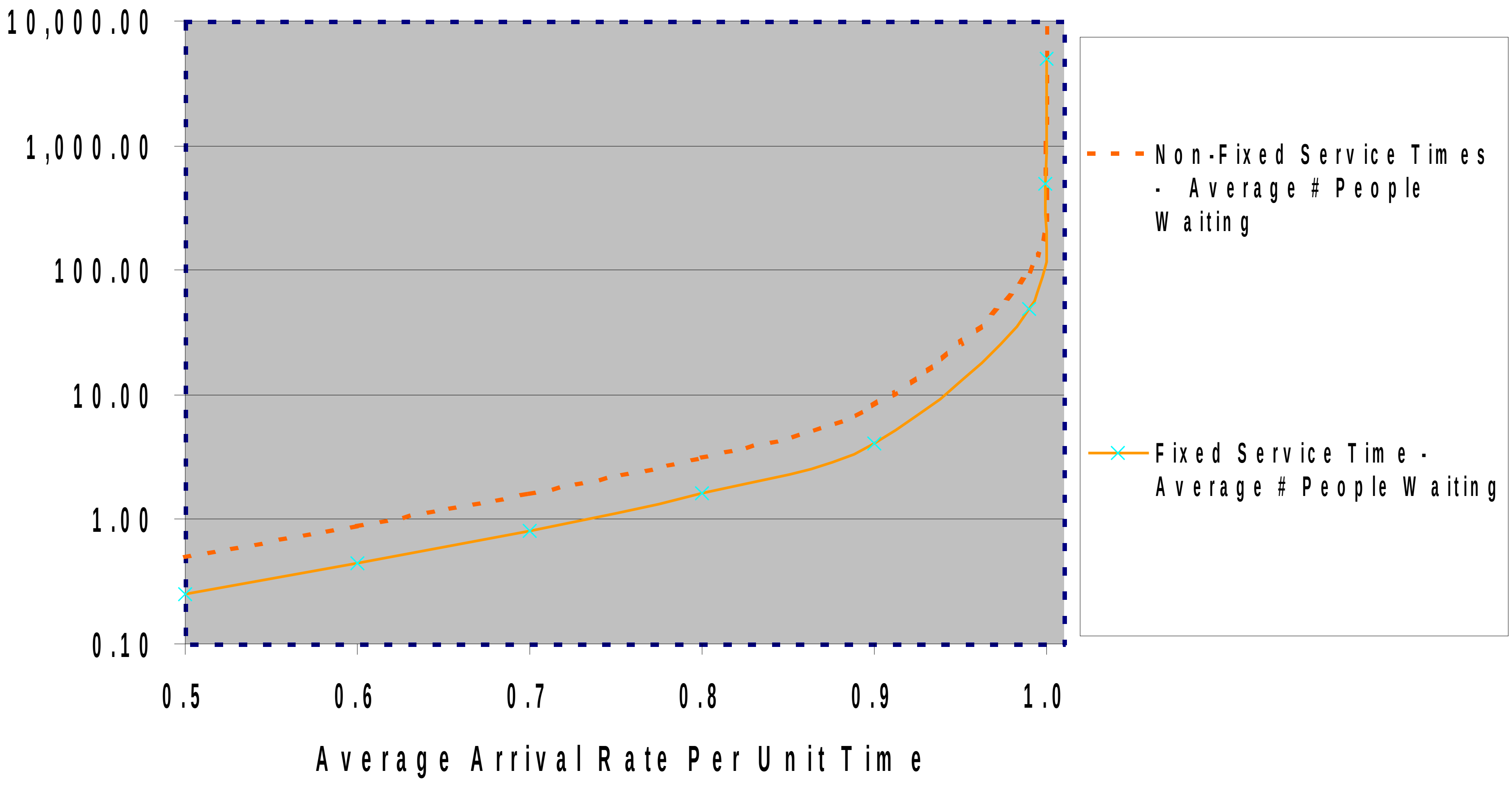


Queuing Theory Insight - Try To Reduce Service Time Variability

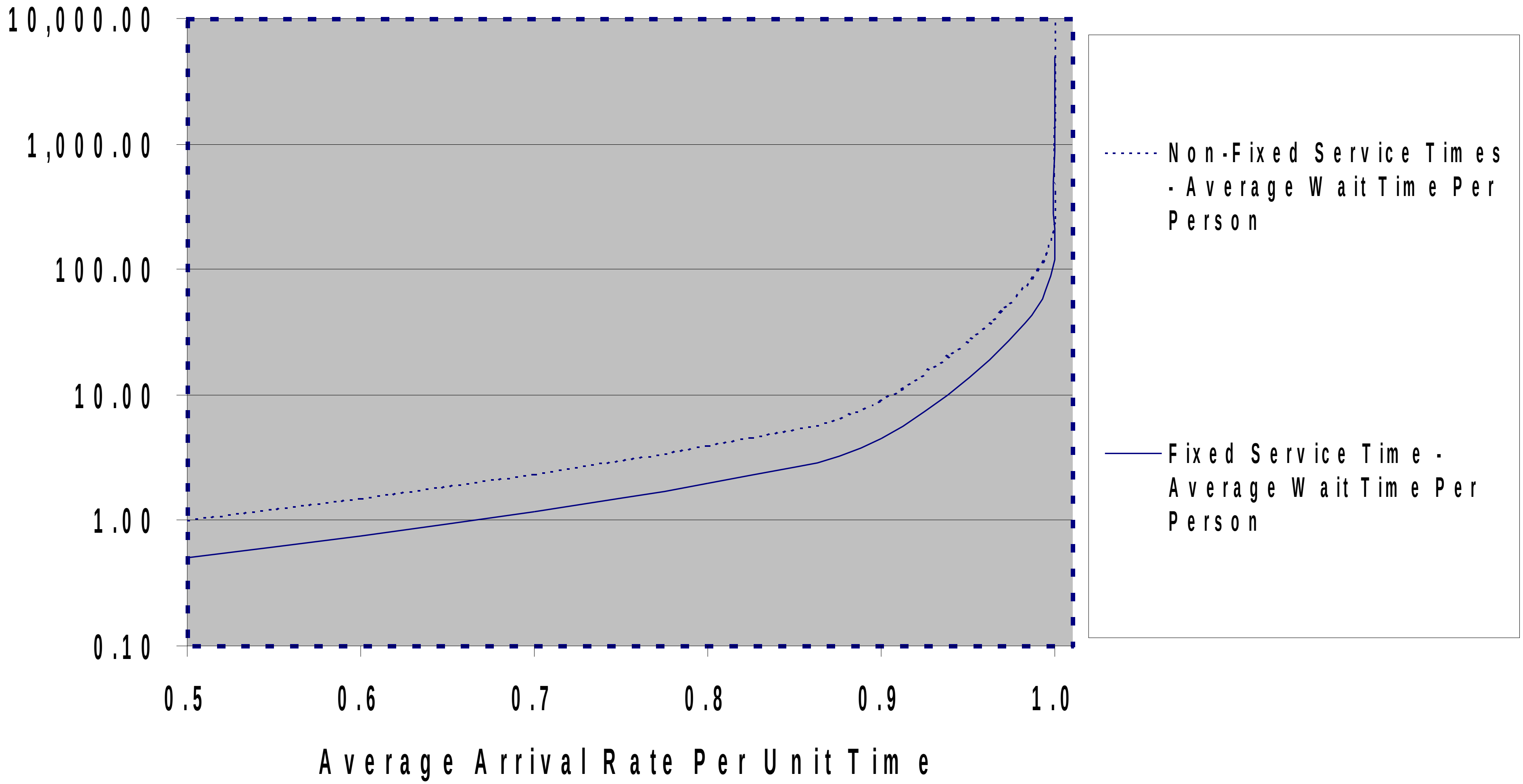
- **Reducing service time variability (when there is 1 server serving patients at an average rate of 1 per unit time and when interarrival times and services times are both exponentially distributed), reduces waiting**

Arrival Rate	Non-Fixed Service Times - Average # People Waiting	Non-Fixed Service Times - Average Wait Time Per Person	Fixed Service Time - Average # People Waiting	Fixed Service Time - Average Wait Time Per Person
0.5000	0.5	1.0	0.3	0.5
0.6000	0.9	1.5	0.5	0.8
0.7000	1.6	2.3	0.8	1.2
0.8000	3.2	4.0	1.6	2.0
0.9000	8.1	9.0	4.1	4.5
0.9900	98.0	99.0	49.0	49.5
0.9990	998.0	999.0	499.0	499.5
0.9999	9998.0	9999.0	4999.0	4999.5

Queuing Theory Insight - Try To Reduce Service Time Variability



Queuing Theory Insight - Try To Reduce Service Time Variability

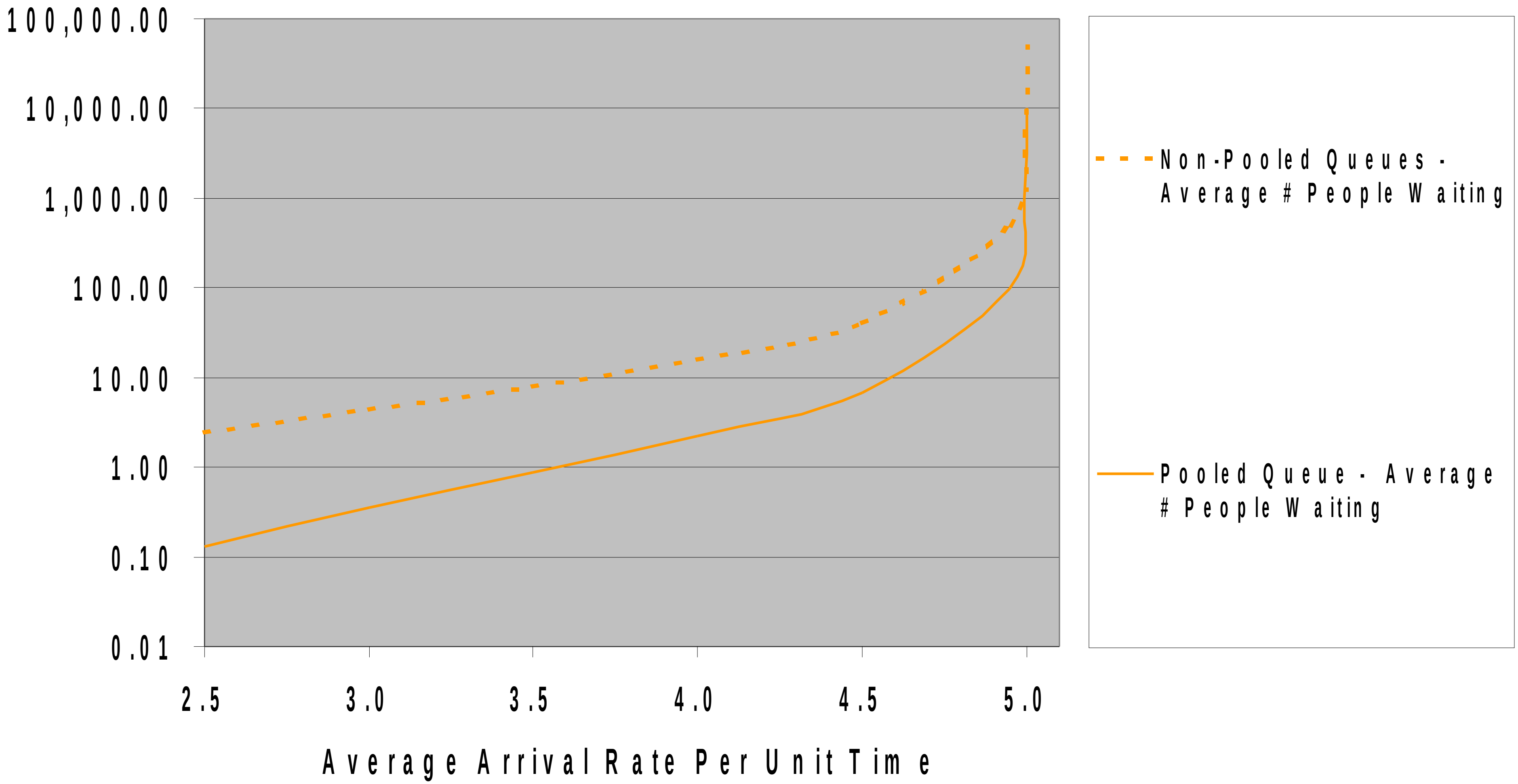


Queuing Theory Insight – Pool Demand & Services

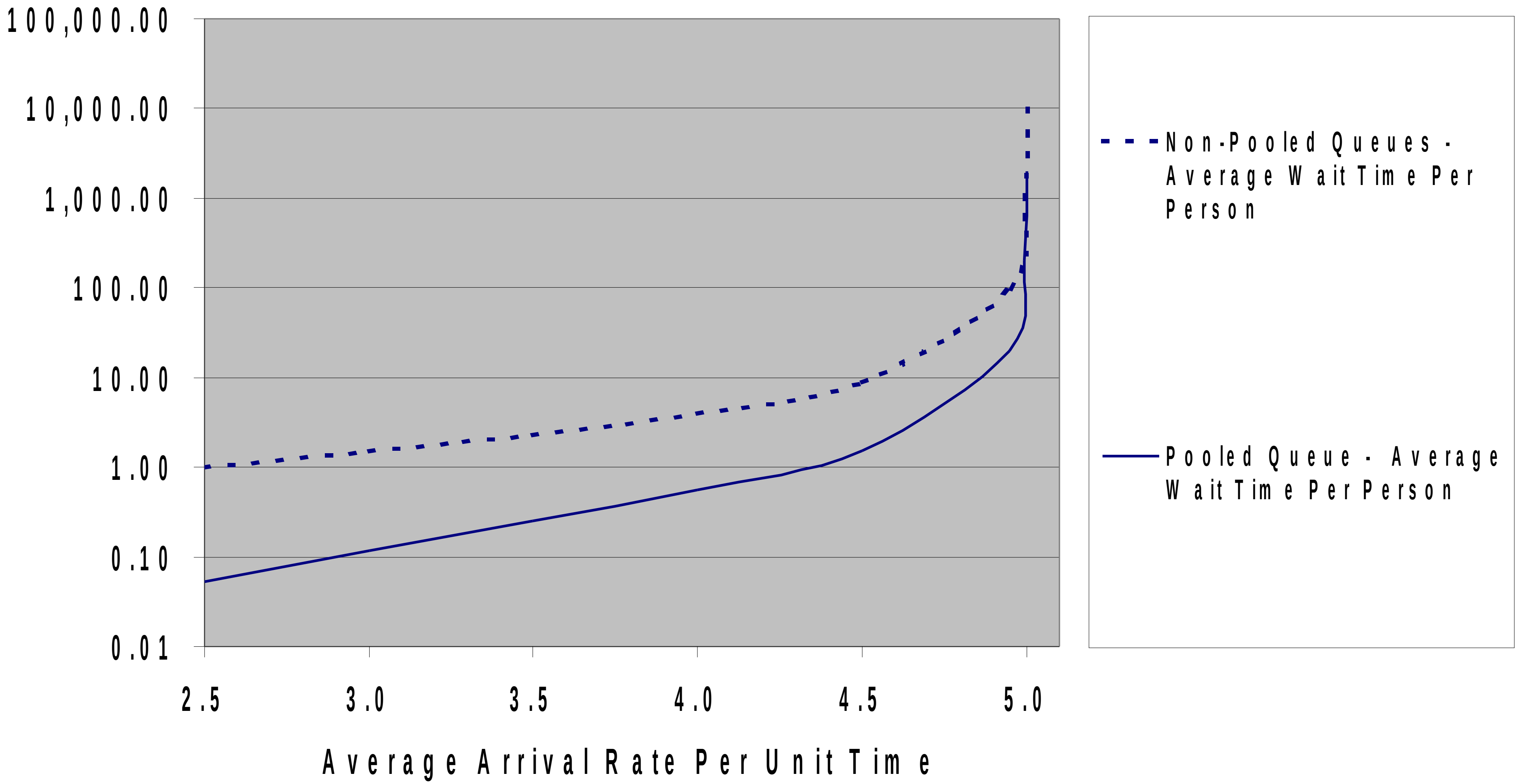
- **When there are 5 servers each serving patients at an average rate of 1 per unit time (and when interarrival times and services times are both exponentially distributed), pooling very significantly reduces waiting**

Arrival Rate	Non-Pooled Queues - Average # People Waiting	Non-Pooled Queues - Average Wait Time Per Person	Pooled Queue - Average # People Waiting	Pooled Queue - Average Wait Time Per Person
2.5000	2.5	1.0	0.1	0.1
3.0000	4.5	1.5	0.4	0.1
3.5000	8.2	2.3	0.9	0.3
4.0000	16.0	4.0	2.2	0.6
4.5000	40.5	9.0	6.9	1.5
4.9500	490.0	99.0	96.5	19.5
4.9950	4,990.0	999.0	996.5	199.5
4.9995	49,990.0	9,999.0	9996.5	1999.5

Queuing Theory Insight – Pool Demand & Services



Queuing Theory Insight – Pool Demand & Services



Queuing Theory Insight – Little's Law

- When the queue has been operating under the same conditions for a long period of time
  - The average # of people being served or waiting for service
  - Equals
    - The average arrival rate
    - Times the average waiting time per person

Queuing Theory Insight – State Dependent Policies

- A particularly powerful result of queuing theory is that queue management can at least sometimes be improved by using policies that are a function of the number and types of waiting patients
  - Dynamic control of patient admissions; this is used to limit admissions when too busy
  - Dynamic staff allocation; this is used to dynamically match capacity to demand since we often cannot limit admissions

Limitations

- **Queuing theory is much harder to apply**
  - **When process characteristics change over time; most of the analysis requires steady state conditions, i.e. conditions that remain the same on the average for a long period of time**
  - **When queues feed other queues, because the arrival and service processes of each queue need to have particular mathematical characteristics for the analysis to be tractable**
  - **When there are lots of individuals with different priorities**

Addressing Those Limitations

- In general, the limitations of queuing theory can be addressed by using computer simulation
- It can handle much more general problems
- Is generally easier to understand
- Can take advantage of lessons learned from queuing theory

Potential Hospital Applications Of Queuing Theory & Simulation

- ICU bed needs analysis
- Pre-surgical screening clinic planning and scheduling optimization
- Evaluating proposed transformational changes
  - Changes in ED
  - Time allocation different patient priorities in imaging and clinics
  - Dynamic allocation of staff
  - . . .

Questions?

Queuing Humour

- **Q: What do you call that line over there?**
  A: A queue.
  Gesundheit!!

- **Q: What do you call a waiting line of little girls' dolls?**
  A: A Barbie-queue.

- **Q: What should you call an advice column for queueing theorists?**
  A: Q-Tips! (Thank you Percy Brill.)

  http://web2.uwindsor.ca/math/hlynka/qfun.html

Thank You